

The Applicability of Markov Analysis Methods to Reliability, Maintainability, and Safety

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Introduction

For many years, Markov models and Markov analysis methods were relegated to that list of exotic but rarely used stochastic modeling techniques, at least for reliability and maintainability purposes. The promulgation of IEC standard 61508 *Functional Safety of Electrical/Electronic/Programmable Electronic Safety-Related Systems* has significantly re-vitalized Markov analysis by requiring the analysis of various disparate failure modes from a safety perspective. The methods also are receiving more attention because today’s software tools make computationally complex Markov analyses easier to perform than in the past.

What is stochastic modeling [1]? A quantitative description of a natural phenomenon is called a mathematical model of that phenomenon. A deterministic model predicts a single outcome from a given set of circumstances; a stochastic model predicts a set of possible outcomes weighted by their likelihoods or probabilities. The word “stochastic” derives from Greek, to aim or to guess, and it means “random” or “chance.” “Sure,” “deterministic,” or “certain” are the antonyms. Such models are to be judged only on the model’s *usefulness for the intended purpose*.

The observer chooses to model a phenomenon as stochastic or deterministic. The choice depends on the observer’s purpose; the criterion for judging this choice is always the model’s usefulness for the intended purpose. To be useful, a stochastic model must reflect all those aspects of the phenomenon under study that are relevant to the question at hand. In addition, the model must allow the deduction of important predictions or implications about the phenomenon.

In reliability, maintainability, and safety (RMS) engineering stochastic modeling is used to describe a system’s operation with respect to time. The component failure and repair times typically become the random variables.

Who was Markov, and What is a Markov Analysis?

Andrei A. Markov graduated from Saint Petersburg University in 1878 and subsequently became a professor there. His early work dealt mainly in number theory and analysis, continued fractions, limits of integrals, approximation theory and the convergence of series. He later applied the method of continued fractions to probability theory.

Markov is particularly remembered for his study of Markov chains. These chains are sequences of random variables in which the future variable is determined by the present variable but is independent of the way in which the present state arose from its predecessors. This work launched the theory of stochastic processes.

Markov analysis looks at a sequence of event and analyzes the tendency of one event to be followed by another. Using this analysis, we can generate a new sequence of random but related events, which appear similar to the original.

The Markov model assumes that the future is independent of the past given the present. When using Markov the random variable is indexed in time, which can be either discrete or continuous.

Many random events are affected by what has happened before. For example, today’s weather does have an influence on what tomorrow’s weather will be. They are not totally independent events.

After observing a long sequence of rainy and sunny days, a Markov model could be used to analyze the likelihood that one kind of weather is followed by another. Assume that 25% of the time, a sunny day follows a rainy day and 75% of the time, rain was followed by more rain. Also assume that sunny days were followed 50% of the time by rain, and 50% by sun.

Using this analysis, one could generate a new sequence of statistically similar weather by following these steps:

1. Start with today's weather.
2. Given today's weather, choose a random number to pick tomorrow's weather.
3. Make tomorrow's weather "today's weather" and go back to step 2.

A result is a particular sequence of days as follows:

Sunny Sunny Rainy Rainy Rainy Rainy Sunny Rainy
Rainy Sunny Sunny...

In other words, the "output chain" would reflect, statistically, the transition probabilities derived from weather that we observed. This stream of events is called a Markov Chain. A Markov Chain, while similar to the source in the micro, is often nonsensical in the macro. This would be a poor way to predict weather because the overall shape of the model has little formal resemblance to the overall form of the source. However, steady-state (long run) probabilities of any day being in a specific state (e.g., rainy or sunny) are the useful and practical result.

Markov Chains, Markov Process, and "semi-Markov" Process Models

There are two basic Markov analysis methods: Markov Chain and Markov Process.

The Markov Chain assumes discrete states and a discrete time parameter; With the Markov Process, states are continuous.

A Markov chain may be described as Homogeneous or Non-Homogeneous. A Homogeneous Markov Chain is characterized by constant transition rates between the states. A Non-Homogeneous Markov Chain is characterized by the fact that the transition rates between the states are functions of a global clock e.g., elapsed mission time.

Markov models are frequently used in RMS work where events, such as the failure or repair of a module, can occur at any point in time. The Markov model evaluates the probability of jumping from one known state into the next logical state (i.e., from everything working to the first item failed, and from the first item failed to the second item's failed state, and so on,) until, depending upon the configuration of the system being considered, the system has reached the final or totally failed state. The basic

assumption of a Markov process is that the behavior of a system in each state is memoryless [2].

A Markov Process is completely characterized by its transition probability matrix.

A memoryless system is characterized by the fact that the future state of the system depends only on its present state. A stationary system is one in which the probabilities that govern the transitions from state to state remain constant with time. In other words, the probability of transitioning from some state i to another state j is the same regardless of the point in time that the transition occurs. The states of the model are defined by system element failures. The transitional probabilities between states are a function of the failure rates of the various system elements. A set of first-order differential equations are developed by describing the probability of being in each state in terms of the transitional probabilities from and to each state. The number of first-order differential equations will equal the number of states of the model. The mathematical problem then becomes one of solving the following equation:

$$\dot{\underline{P}} = [A]\underline{P}$$

Where $\dot{\underline{P}}$ and \underline{P} are $n \times 1$ column vectors, $[A]$ is an $n \times n$ matrix and n is the number of states in the system. The solution of this equation is:

$$\underline{P} = \exp[A]t \cdot \underline{P}(0)$$

Where $\exp[A]t$ is an $n \times n$ matrix and $\underline{P}(0)$ is the initial probability vector describing the initial state of the system. Two methods that are particularly well suited for the digital computer for computing the matrix $\exp[A]t$ are the infinite series method and the eigenvalue/eigenvector method. Figure 1 presents a flow chart that illustrates the procedure used to develop a Markov model.

Repairs can be addressed by using repair rates that account for the return from any given failed state to the proceeding working state. All of this results in a complex diagram of bubbles, representing each state, and directed lines, with arrows, showing the movement from one state to the next, or to the preceding state. As the Markov Diagram is drawn, (see subsequent example) the failure rate values and the repair rate numbers can be entered into an $n \times n$ matrix (where "n" is the number of states being considered) commonly called the "transition matrix".

Figure 2 shows a simple state transition or bubble diagram (a diagram with two states: A = operational, B = failed) with a failure rate λ and a repair rate μ (for the homogeneous case, for the non-homogeneous case $\lambda(t)$ and $\mu(t)$.) Movements from left to right indicate failure and movement from right to left indicates recovery.

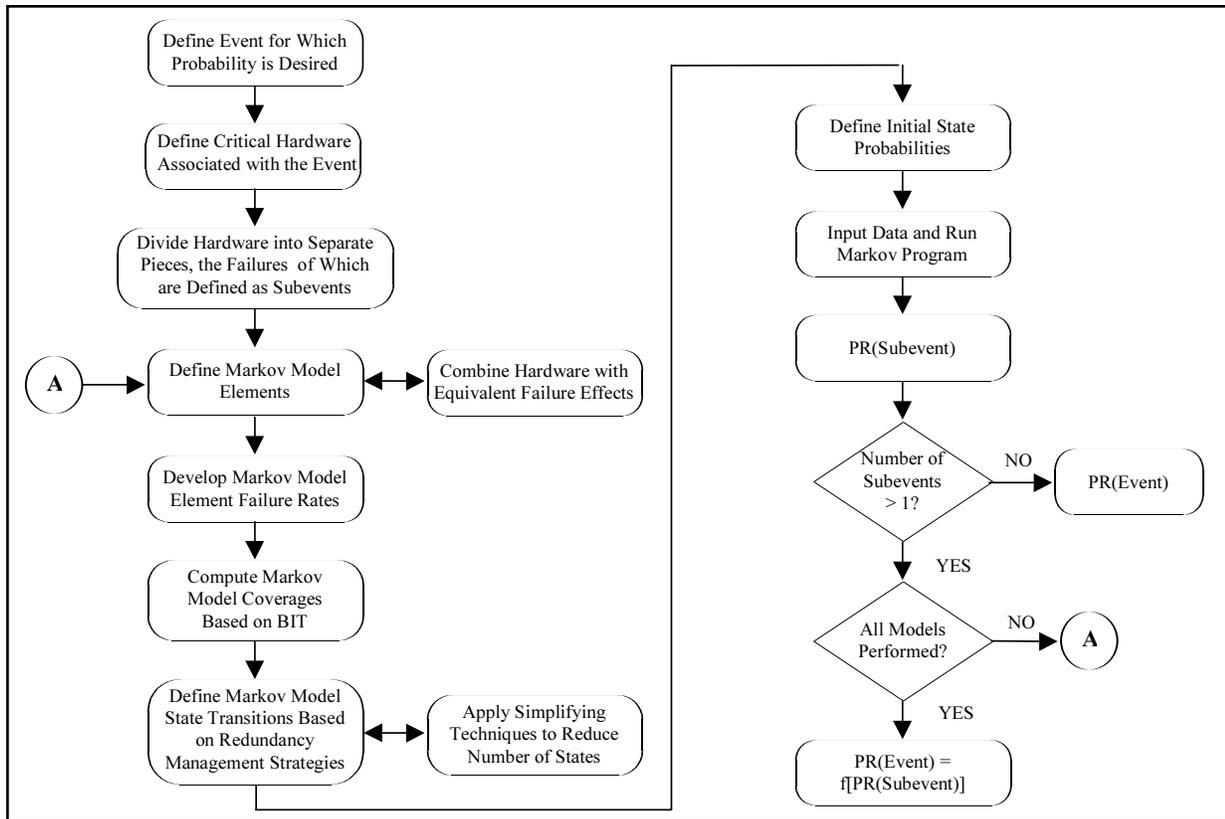


Figure 1. Markov Modeling Process

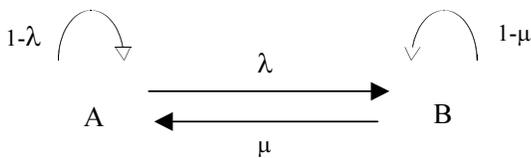


Figure 2. Simple State Transition Diagram

Today, user-friendly computer programs can solve the transition matrix, and thereby, develop the long run or steady-state probabilities i.e., the probability that the system is in any of the defined states.

Also, if we make the step (Δt) small enough we can approximate a Markov Process by a Markov Chain.

Semi-Markov process models are also frequently used in reliability theory. The semi-Markov model is a probabilistic model useful in analyzing complex dynamical systems [3]. Its behavior is similar to that of a pure Markov model. With semi-Markov models, however, the transition times and the probabilities (distributions) depend on the time at which the system reached the present state. This means that the transition rates in a particular state depend on the time already spent in that state (sojourn time) but that they do not depend on the path by which the present state

was reached. Thus transition distributions in the semi-Markov process can be non-exponential.

The semi-Markov theory involves the concepts of state and state transition.

The most important statistics of the semi-Markov process are the interval transition probabilities.

Reliability, Maintainability, and Safety Application of Markov Analysis Methods: the Pros and Cons

Markov methods can be powerful tools in RMS engineering. Markov chains are commonly used to study the dependability of complex systems. Markov analysis provides a means of analyzing the RMS of systems whose components exhibit strong dependencies. Other systems analysis methods (e.g., fault tree analysis) often assume total component independence. Used alone, these methods may lead to optimistic predictions for the system reliability and safety parameters.

The exponential distribution i.e., a constant failure rate, is a common assumption in system analyses based upon Markov methods.

Markov methods offer significant advantages over other reliability modeling techniques, some of these advantages are:

1. **Simplistic modeling approach:** The models are simple to generate although they do require a more complicated mathematical approach.
2. **Redundancy management techniques:** System reconfiguration required by failures is easily incorporated in the model.
3. **Coverage:** Covered and uncovered failures of components are mutually exclusive events. These are not easily modeled using classical techniques, but are readily handled by the Markov mathematics.
4. **Complex systems:** Many simplifying techniques exist which allow the modeling of complex systems.
5. **Sequenced events:** Often the analyst is interested in computing the probability of an event resulting from a sequence of sub-events. While these types of problems do not lend themselves well to classical techniques, they are easily handled using Markov modeling.

The advantage of the Markov process is that it neatly describes both the failure of an item and its subsequent repair. It develops the probability of an item being in a given state, as a function of the sequence through which the item has traveled. The Markov process can thus easily describe degraded states of operation, where the item has either “partially” failed or is in a degraded state where some functions are performed while others are not. Competing techniques, e.g., FMEA and Fault Tree Analysis (FTA), have a difficult time dealing with degraded states as contrasted with outright failures.

The major drawback of Markov methods is the explosion of the number of states as the size of the system increases. The resulting diagrams for large systems are generally extremely large and complicated, difficult to construct and computationally extensive.

Sometimes, however, a combination approach is best. Markov models may be used to analyze smaller pseudo-systems with strong dependencies requiring accurate evaluation. Then other analysis techniques, such as FTA, may be used to evaluate the total system using simpler probabilistic calculation techniques. Large systems, which exhibit strong component dependencies in isolated and critical parts of the system, may thus be analyzed using a combination of Markov analysis and simpler quantitative models. For example, if a small dynamic portion is solved separately and then combined with the overall static consequences the resulting Markov model can be much smaller.

An Example of the Development of a Markov Model

In order to illustrate how the Markov model equations are developed, assume we have a system that is made up of two elements. Each element has two mutually exclusive states – a good and a failed state. The states of the model are generated based on the

elements being in one of these two states. The probabilities that cause transition from state to state are a function of the element failure rates. An element with constant failure rate (1) has a transitional probability that is approximated by $\lambda \cdot \Delta t$. The probability of more than one element failure in Δt is considered negligible. A flow diagram of this two-element problem is presented in Figure 3.

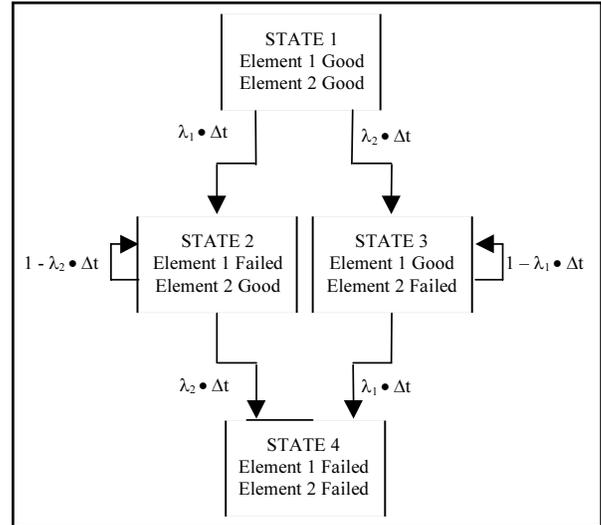


Figure 3. Markov Flow Diagram

We develop the Markov differential equation by describing the probability of being in each of the states at time $t + \Delta t$ as a function of the state of the system at time t . The probability of being in state 1 at some time $t + \Delta t$ is equal to the probability of being in state 1 at time t and not transitioning out during Δt . This can be written as:

$$P1(t + \Delta t) = P1(t) \cdot [1 - (\lambda_1 + \lambda_2) \cdot \Delta t]$$

The probability of being in state 2 at time $t + \Delta t$ is equal to the probability of being in state 1 at time t and transitioning to state 2 in Δt plus the probability of being in state 2 at time t and **not** transitioning out during Δt . This can be written as:

$$P2(t + \Delta t) = P1(t) \cdot \lambda_1 \cdot \Delta t + P2(t)(1 - \lambda_2 \cdot \Delta t)$$

The other state probabilities are generated in the same manner resulting in the following equations:

$$P1(t + \Delta t) = P1(t) \cdot [1 - (\lambda_1 + \lambda_2) \cdot \Delta t]$$

$$P2(t + \Delta t) = P1(t) \cdot \lambda_1 \cdot \Delta t + P2(t)(1 - \lambda_1 \cdot \Delta t)$$

$$P3(t + \Delta t) = P1(t) \cdot \lambda_2 \cdot \Delta t + P3(t)(1 - \lambda_1 \cdot \Delta t)$$

$$P4(t + \Delta t) = P2(t) \cdot \lambda_2 \cdot \Delta t + P3(t) \cdot \lambda_1 \cdot \Delta t + P4(t)$$

Rearranging the equations:

$$\begin{aligned} [P1(t + \Delta t) - P1(t)]/\Delta t &= -(\lambda_1 + \lambda_2) \cdot P1(t) \\ [P2(t + \Delta t) - P2(t)]/\Delta t &= \lambda_1 \cdot P1(t) - \lambda_2 \cdot P2(t) \\ [P3(t + \Delta t) - P3(t)]/\Delta t &= \lambda_2 \cdot P1(t) - \lambda_1 \cdot P3(t) \\ [P4(t + \Delta t) - P4(t)]/\Delta t &= \lambda_2 \cdot P2(t) + \lambda_1 \cdot P3(t) \end{aligned}$$

Taking the limit as $\Delta t \rightarrow 0$:

$$\begin{aligned} dP1(t)/dt &= -(\lambda_1 + \lambda_2) \cdot P1(t) \\ dP2(t)/dt &= \lambda_1 \cdot P1(t) - \lambda_2 \cdot P2(t) \\ dP3(t)/dt &= \lambda_2 \cdot P1(t) - \lambda_1 \cdot P3(t) \\ dP4(t)/dt &= \lambda_1 \cdot P2(t) + \lambda_2 \cdot P3(t) \end{aligned}$$

In matrix form this becomes:

$$\begin{bmatrix} dP1(t)/dt \\ dP2(t)/dt \\ dP3(t)/dt \\ dP4(t)/dt \end{bmatrix} = \begin{bmatrix} -(\lambda_1 + \lambda_2) & 0 & 0 & 0 \\ \lambda_1 & -\lambda_2 & 0 & 0 \\ \lambda_2 & 0 & -\lambda_1 & 0 \\ 0 & \lambda_2 & \lambda_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P1(t) \\ P2(t) \\ P3(t) \\ P4(t) \end{bmatrix}$$

Or $\dot{\underline{P}} = [A] \cdot \underline{P}$ where $[A]$ is defined as the state transition matrix. The important thing to note here is that the analyst need only generate the states and the transitions between states as defined by the element failure rates. This information can then be inputted to the computer in a form that allows it to set up the state transition matrix and compute the state probabilities using matrix mathematics.

Markov Model Reduction Techniques

Since the Markov modeling approach can generate all the possible states of a system, the number of states can be extremely large even for a relatively small number of Markov elements. Thus, the analyst using the Markov modeling approach must become familiar with reduction techniques that can be applied to reduce the number of states significantly while maintaining the accuracy of the model. For example, if we assume a system has 10 elements, each of which has two states (good and failed), the total number of possible states becomes:

$$\# \text{ states} = 2^n = 2^{10} = 1024$$

Furthermore, in modeling a modern digital system, a system containing only 10 elements would be considered small. Fortunately, simplification techniques can be used alone or in combination to reduce the number of states in the model.

One approach is to use the principle, that states that represent multiple levels of failure contribute insignificantly to the overall probability of failure of the system. The model then can be truncated at a certain failure level, combining all states below that level into one failed state. If for instance it is desired to truncate at the n^{th} level, all state transitions from $n - 1$ level states would

be directed to one n^{th} order failed state. Care should be taken, however, to make sure that this truncation does not have an overly conservative impact on the system.

Many elements have the same, or nearly the same, impact on system operation when failed, so the states that are the result of failure of these elements can be combined. As an example, assume we have a two-channel control in dual active mode. Dual active mode means both channels are simultaneously in control. Let each channel have a failure rate λ . If one channel fails, we can control with the other good channel. Because loss of either channel leads to the same effect (i.e., single channel control), the two corresponding states can be combined. Figure 4 shows the Markov model for this system with no reduction, and the equivalent, reduced model by combining States 2 and 3. Because the system impact is the same independent of which channel failed first, and because the channel failure rates are the same, we can reduce the number of states with no loss of accuracy. If independence is not the case, assumptions have to be made as to which element failure caused transition to the new state so that a conservative approximation to the transitions out of the state can be made.

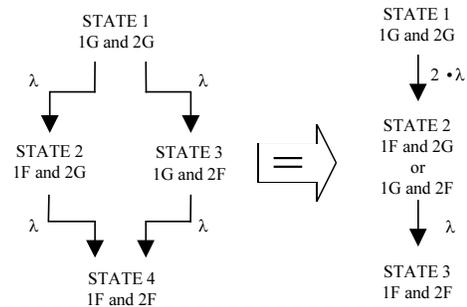


Figure 4. Two Channel Example

Often, the failure of a certain element causes the loss of other elements in a system. An example could be the loss of a power supply. In this case, the analyst need not define transitions for the other lost element(s) because by definition, they also are no longer part of the functioning system.

Another reduction technique involves dividing the top-level event for which the probability of failure is desired into n sub-events, each of which is modeled separately. The probabilities for each sub-event are then combined to yield the probability of the top-level event. If for instance the system being modeled has ten elements, we have a possible of 1024 total states. If the top-level event containing these ten elements can be broken down into two sub-events, each containing five elements, the system can be described with two models each with a possible thirty-two states. If the top level event has probability P and the two sub-events have probabilities $P1$ and $P2$ respectively, the top level probability can be computed as $P = f(P1,P2)$.

Application of Coverage to Markov Modeling

In redundant system modeling, we might consider three Markov element states – good, failed covered, and failed uncovered. Covered and uncovered Markov element states are mutually exclusive meaning that an element cannot fail both covered and uncovered. System coverage is generally defined in terms of the conditional probability: $P[\text{detect, isolate, reconfigure} | \text{failure}]$.

When computing coverages for Markov model elements, we are concerned with that portion of the Markov element failure rate that is detectable and isolatable. Reconfiguration then becomes a function of the resources available at the time of failure.

As an example of how coverage is used in the Markov model, we will return to the two channel dual active control discussed previously. In this case if either channel fails **covered**, the other channel has the ability to take over full control. However, if either channel fails **uncovered**, system failure occurs. The Markov model for this example appears in Figure 5. Note that once state two is entered, no resources are available and both the covered and uncovered portions of the remaining channels failure rate are routed to system failure.

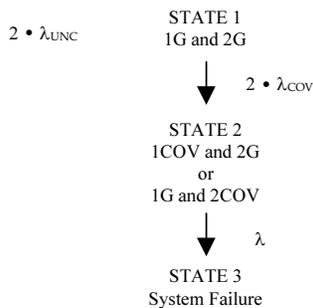


Figure 5. Coverage Example

International Standards and Markov Analysis

There are at present two international standards dealing with the Markov approach. They are IEC 61165 and IEC 61508. IEC 61165 provides guidance on the application of Markov techniques to dependability analysis. IEC 61508 has re-vitalized the need for Markov analysis due to the standard's requirement to analyze disparate failure modes, ranging from Failed Safe, Detected down to Failed Dangerous, Undetected. The standard itself is broken down into seven different parts. The main goal of IEC 61508 [5] is to estimate the probability of failure on demand for the critical system being examined.

A brief description of these two IEC standards including the seven different parts of IEC 61508 follows.

IEC 61165 – Application of MARKOV Techniques.

This standard provides guidance on the application of

Markov techniques to dependability analysis. Several distinct analytical methods of dependability analysis are available; of which Markov analysis is one method. IEC 300-3-1 gives an overview of available methods and their general characteristics. The relative merits of various methods and their individual or combined applicability in evaluating the dependability of a given system or component should be carefully examined by the analyst prior to deciding on the use of Markov analysis.

IEC 61508 – Functional safety of electrical/electronic/programmable electronic safety-related systems – Part 1: General requirements.

This document sets out a generic approach for all safety lifecycle activities for systems comprised of electrical and/or electronic and/or programmable electronic components (electrical/electronic/programmable electronic systems (E/E/PESs)) that are used to perform safety functions. This unified approach has been adopted in order that a rational and consistent technical policy be developed for all electrically-based safety-related systems. It is intended to facilitate the development of application sector standards. It has the status of a basic safety publication in accordance with IEC Guide 104.

IEC 61508 – Functional safety of electrical/electronic/programmable electronic safety-related systems – Part 2: Requirements for electrical/electronic/programmable safety-related systems. This document is intended to be used only after a thorough understanding of IEC 61508-1, which provides the overall framework for the achievement of functional safety.

IEC 61508 – Functional safety of electrical/electronic/programmable electronic safety-related systems – Part 3: Software requirements. This document applies to any software forming part of a safety-related system or used to develop a safety-related system within the scope of IEC 61508-1 and -2. It provides requirements: a) for safety lifecycle phases and activities; b) for information relating to the software safety validation; c) for the preparation of information and procedures concerning software; d) to be met by the organization carrying out modifications to safety-related software; and e) for supporting tools.

IEC 61508 – Functional safety of electrical/electronic/programmable electronic safety-related systems – Part 4: Definitions and abbreviations. This document contains the definitions and explanation of terms used in parts 1 through 7 of this standard. It is intended for use by technical committees in the preparation of standards in accordance with the principles contained in IEC Guide 104 and ISO/IEC Guide 51.

IEC 61508 – Functional safety of electrical/electronic/programmable electronic safety-related systems – Part 5: Examples of methods for the determination of safety integrity levels. This document provides information on the underlying concepts of risk and the relationship of risk to safety integrity; a number of methods that will enable the safety integrity levels (SIL) for the E/E/PE safety-related systems, other technology safety-related systems and external risk reduction facilities to be determined. It is intended for use by technical committees in the preparation of standards in accordance with the principles contained in IEC Guide 104 and ISO/IEC Guide 51.

IEC 61508 – Functional safety of electrical/electronic/programmable electronic safety-related systems – Part 6: Guidelines on the application of IEC 61508-2 and -3. This document contains information and guidelines on IEC 61508-2 and IEC 61508-3.

IEC 61508 – Functional safety of electrical/electronic/programmable electronic safety-related systems – Part 7: Overview of techniques and measures. This document contains an overview of various safety techniques and measures relevant to IEC 61508-2 and IEC 61508-3.

Examples of Computerized Markov Analysis Tools

Because of the complexity and computational difficulty of Markov methods, computer solutions are essentially mandatory. There are a number of computerized Markov tools available to perform these types of analyses. A brief discussion of two of these tools follows.

SHARPE – Symbolic Hierarchical Automated Reliability and Performance Evaluator [6]. This is a tool for specifying and analyzing performance, reliability and performability models. It is a toolkit that provides a specification language and solution methods for most of the commonly used model types for performance, reliability, and performability modeling. Model types include combinatorial one such as fault-trees and queuing networks and state-space ones such as Markov and semi-Markov reward models as well stochastic Petri nets. Steady-state, transient and interval measures can be computed. Output measures of a model can be used as parameters of other models. This facilitates the hierarchical combination of different model types. To increase the usability of this modeling tool, a graphical user interface (GUI) for SHARPE has been implemented. The SHARPE GUI implements eight interchangeable modeling description techniques for reliability engineering: fault trees, Markov chains, reliability block diagrams, reliability graphs, generalized stochastic Petri nets, product queuing networks, multi-chain product form queuing networks and task graphs. In the future, all the modeling description techniques contained in

SHARPE will be available in the GUI (phase mission, multi-components fault trees, semi-Markov chains). The hierarchy feature is also implemented in the GUI. Java is the language chosen for GUI implementation.

MKV Version 3.0 – Markov Analysis Software. This program also analyses non-homogeneous processes by allowing time-dependent transition rates to be defined. Systems with time-dependent transition rates are strictly non-Markovian, however the addition of this facility in the MKV program allows certain types of aging processes to be modeled. MKV calculates a wide range of system parameters during the integration process. These parameters are Unavailability, Availability, Unreliability, Reliability, Failure frequency (unconditional failure intensity), Repair frequency (unconditional repair intensity), Conditional failure intensity, Conditional repair intensity, Number of expected failures, Number of expected repairs, Mean unavailability over lifetime, Mean availability over lifetime, Expected total downtime over lifetime and Expected total uptime over lifetime. Markov analysis provides a means of analyzing the reliability and availability of systems whose components exhibit strong dependencies. Other systems analysis methods (such as the Kinetic Tree Theory method generally employed in fault tree analysis) often assume component independence, which may lead to optimistic predictions for the system availability and reliability parameters.

Summary

Combinatorial models such as reliability block diagrams and fault trees are frequently used to predict the reliability, maintainability, and safety of complex systems. Unfortunately, these methods cannot accurately model dynamic system behavior. Because of its unique ability to handle dynamic cases, Markov analysis can be a powerful tool in the RMS analyses of dynamic systems. A Markov model breaks the system configuration into a number of states. Each of these states is connected to all other states by transition rates. It then utilizes transition matrixes and state transition diagrams to represent the various system states and the possible transitions between these states. These state diagrams are more visual in nature than mathematical representations thus they are much easier to interpret. These advantages, however, must always be traded-off against the explosion in complexity and resulting increase in computation time as the number of states increase.

Thus Markov analysis can be a powerful RMS analysis tool. It allows the analyst to model complex, dynamic, highly distributed, fault tolerant systems that would otherwise be very difficult or impossible to model with classical techniques. Markov techniques decrease the analyst's task by reducing the problem from one of mathematical computation to that of state modeling. Model reduction techniques also exist which can yield relatively simple models with insignificant impact on model accuracy. In many situations it is also possible to use a synergistic combination of Markov analysis and more conventional approaches, e.g.,

reliability block diagrams, fault trees, etc. in such a way that the resulting individual dynamic portion(s) are relatively small and tractable.

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Other START Sheets Available

Many Selected Topics in Assurance Related Technologies (START) sheets have been published on subjects of interest in reliability, maintainability, quality, and supportability. START sheets are available on-line in their entirety at <<http://rac.alionscience.com/DATA/START>>.

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