START Selected Topics in Assurance Related Technologies

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Design of Experiments

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Introduction

The design of experiments (DOE), also called the statistical design of experiments, is a tool for determining the significance of different factors affecting process quality, and for calculating optimal settings for controllable factors. For example, we may believe that operating temperature and wave height affect the number of defects from a wave solder machine. DOE provides a fast and efficient means for determining the values of these parameters that would produce the fewest number of defects.

DOE was pioneered by Sir Ronald Fisher as an improvement on conventional experimental techniques for use in agricultural applications, such as improving yield. More recent contributors to DOE theory have been George Box, Dorian Shainan, and Genichi Taguchi, among others. While it is possible to perform and analyze experiments in other ways, this discussion will consider experiments using orthogonal arrays (to be defined later) and limited to two different settings of each factor tested.

The DOE process to be discussed consists of seven steps, as shown in Table 1.

Table 1. DOE Process

1. Select factors to be tested and a measure of process outcome
2. Select test settings for each factor
3. Select an appropriate orthogonal array
4. Run the tests
5. Analyze the results
6. Calculate optimum settings for each factor
7. Run confirmation test(s)

Selecting Factors

It is not always obvious what factors are of interest. To minimize the number of tests run, a good approach is to convene a team familiar with the process to be tested and have them brainstorm and rank the results to produce a short list of factors to test. A measure of process output must also be identified. This measure could be the defect rate for a wave solder process, horsepower for an engine, etc. It must be a parameter we want to optimize.

Selecting Test Settings

Under a self-imposed limitation of testing only two values, we select a high and low value for each of the factors to be tested. These values must be far enough apart to make a difference and close enough together so that we can assume a linear change in the test outcome as each factor varies between high and low values.

It does not matter which value is designated "high" or "low"; the analysis will produce the same results. A factor such as the presence or absence of flux is handled by designating one of these conditions as "high" and the other as "low." To facilitate analysis the "high" factor is coded as plus one and the "low" factor as minus one. For example, if we select operating temperature as a factor and wish to test at 200 degrees and 400 degrees, one of these values is coded plus one and the other minus one. If 400 degrees is plus one and, after test analysis and optimization, we find the best temperature to use is a value coded between the two test factors, we will convert its value linearly to an actual temperature. A coded value of zero would be converted to 300 degrees. A code value of plus 0.5 would be translated to 350 degrees, etc.

Selecting an Orthogonal Array

An orthogonal array is a matrix of test settings designed to facilitate separation of the effects of each factor on the experiment outcome. Table 2 shows an orthogonal array testing the effects of temperature and wave height on the defect rate of a wave solder process.

Table 2. Orthogonal Array

Test Set Up	Temperature T	Wave Height W	Interaction (T*W)	Defect Rate F
1	-1	-1	+1	f1
2	+1	-1	-1	f2
3	-1	+1	-1	f3
4	+1	+1	+1	f4
AVG -	(f1+f3)/2	(f1+f2)/2	(f2+f3)/2	
AVG +	(f2+f4)/2	(f3+f4)/2	(f1+f4)/2	
D	(f2+f4)/2 - (f1+f3)/2	(f3+f4)/2 - (f1+f2)/2	(f1+f4)/2 - (f2+f3)/2	

Four tests are run. The first with temperature set to the low value (represented by -1 in the matrix) and wave height set to its low value (also represented by -1 in the matrix). The resulting defect rate is recorded. The interaction column is a by-product of the temperature and height setting that will measure any effects of interaction between the two factors. Test 2 is run with temperature set to its high value and wave height to its low value. In test 3, temperature is low and wave height high. In test 4, both are high.

D is the difference between the average defect rate when a factor is set to its high value and the average results when it is set to its low value, computed for each factor: temperature, wave height and the interaction.

Orthogonal Arrays always have 2n rows, where n is the number of factors to be tested. The number of columns for factors and interactions will be one less than the number of rows.

Running the Tests

Ideally, the tests represented by the rows of the orthogonal array should each be run several times and the results averaged. The order of the tests should be randomized so that any unknown factor influencing the outcome will not bias the results. The effects of uncontrollable factors, such as atmospheric pressure, can be averaged out by scheduling tests so that they are run under differing levels of the uncontrolled factor. Untested controllable factors, such as machine speed, should be standardized for the tests.

Analyzing the Results

Table 2 contains all the information needed to calculate an expected defect rate for any combination of temperature and wave height between the high and low values. It will be given by the linear regression equation:

$$f = (f1+f2+f3+f4)/4 + (D_T)T/2 + (D_W)W/2 + (D_{T*W})(T*W)/2$$

Where the first term is the average result of the experiment, D_T is the calculated value of D for temperature, T is temperature, etc. Note that T, W and (T*W) are the coded values. Thus, to find f for any combination of temperature and wave height, the actual values of interest must be coded.

Optimization

Once the regression equation is determined, settings for each factor can be determined to optimize the expected outcome of the process. To illustrate, assume we have run an experiment and have the results given in Figure 1.

Run	A	B	A*B	Y
1	-	-	+	10
2	+	-	-	6
3	-	+	-	8
4	+	+	+	4
Avg -	9	8	7	$\bar{Y} = 7$
Avg +	5	6	7	
Δ	-4	-2	-	

Y = 7 - 2A - B

Figure 1. Sample Test Results

Figure 1 shows that the interaction A*B has no effect on the results, so factors can be adjusted without concern for interactions.

If the sample data was a test of the wave solder process, with A being the temperature, and B being the wave height, and the output, Y, being defect rate, we would want to set the factors to the values which would give the lowest outcome. In this case, setting both A and B to plus one would give the lowest value of Y, and hence we would adjust our process to use the high values of temperature and wave height. If the data was from an experiment where the output was a parameter we wanted to maximize, such as fuel economy, we would set factors A and B, whatever they represent, to their low values. If the output was something that we wanted to fix at some level, such as the thickness of a glass passivation layer that we wanted to fix at five microns, we could set factor A to its high value and B to the value coded as zero (and other ways). We should note:

- Factors such as the presence or absence of flux can be set at only the values coded as plus one (present) or minus one (absent).
- Some factors may be easier or less costly to adjust than others. This fact should be considered.
- Extrapolations beyond the ranges used in the test (the values coded as plus one and minus one) necessarily assume the factor's impact will remain linear beyond the region tested. This assumption can be dangerous.

Confirmation

The final step in the DOE process is to confirm the optimization results by running another test at the optimized settings to see if the expected results do indeed result. It is always possible that some factor not tested has a significant impact on the outcome. A confirmation test with unexpected results indicates more thought is required.

Saturated Arrays

There are occasions when interactions may be ignored, perhaps, for example, when testing fuel economy using tire pressure and fuel-air mixture as factors. In such cases, additional factors can be tested without adding rows or columns to the test matrix, using what are called saturated arrays. In a saturated array, a controllable factor is substituted for the by-product interactions. (Note: it is assumed that the additional factors are also not likely to interact with any of the other factors.) Using a saturated array, the two-factor test matrix shown in Table 2 and Figure 1 could be used to test three factors as shown in Table 3. Factor C, a controllable factor, replaces the by-product interaction A*B. Using the saturated array allows three factors to be tested in four tests rather than in eight, as would be required by a standard orthogonal array.

Robust Design

Robust designs are those that will perform satisfactorily under all expected conditions. Returning to our wave solder example, suppose we have decided that temperature, wave height and solder composition are the most important controllable factors. We may also feel that the size and the number of layers of the printed circuit boards going through the wave solder machine are also important, but we have no control of these factors. To determine the settings that provide the lowest defect rate under all circumstances, we would run the experiment four times as shown in Table 4. Our controlled factors are designated A, B and C, and interactions are not shown. The two uncontrolled factors are labeled D and E. In this case we could likely obtain suitable samples for the test. If the uncontrolled factors were, say, humidity and atmospheric pressure, we would have to wait for nature

Table 3. Saturated Array

Test	A	B	C	Outcome
1	+	+	+	
2	-	+	-	
3	+	-	-	
4	-	-	+	

Table 4. Testing for Robustness

Controlled Factors Settings					Uncontrolled Factors "Settings"			
Test Run	A	B	C	Uncontrolled Factors D E	- -	+ -	- +	+ +
1	-	-	-		9.0	1.8	1.6	7.8
2	+	-	-		2.6	2.0	2.3	4.8
3	-	+	-		2.7	2.0	1.5	3.4
4	+	+	-		2.2	1.5	1.7	1.9
5	-	-	+		2.4	1.6	1.5	2.9
6	+	-	+		1.9	1.7	1.7	1.8
7	-	+	+		3.3	1.6	1.6	3.3
8	+	+	+		2.6	1.8	1.6	1.9

to provide the desired "settings." The set of controlled factors are called the inner array and the uncontrolled factors called the outer array.

Analysis of Table 4 shows that the settings of test run 6 would provide the lowest defect rate under all the uncontrolled conditions, even though better outcomes were produced by some test settings for specific combinations of the uncontrolled factors. If all the combinations of uncontrolled factors were equally likely, the controlled factors should be set to the values used in test run 6.

For Further Study

A good starting text is Understanding Industrial Designed Experiments, by S.R. Schmidt and R.G. Launsby, Air Academy Press, Colorado Springs, CO, 1989. Other references are:

Quality by Experimental Design, by T.B. Barker, Marcel Dekker, New York, NY, 1985.

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Statistical Tables for Biological, Agricultural and Medical Research, by R.A. Fisher and F. Yates, Oliver & Bays Ltd, London, 1953.

Fundamental Concepts in the Design of Experiments, by C.R. Hicks, Holt, Rinehart and Winston, New York, NY, 1982.

Introduction to Quality Engineering, by G. Taguchi, American Supplier Institute, Dearborn, MI, 1986.

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